Babasaheb Bhimrao Ambedkar Bihar University, Muzaffarpur
Directorate of Distance Education
T.D.C. 4th Semester Examination
Subject:- Mathematics (Hons.)
Paper – 4th
Model Paper (Full Marks – 80)

Answer any four questions:-
किन्हीं चार प्रश्नों के उत्तर दें।

1 (a) Define Scalar Triple Product of three non-zero vectors and interpret it geometrically.

(b) Define Vector Triple Product and Establish the formula $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

2 Prove that:

(a) $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

(b) $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

3 (a) If $\vec{a}, \vec{b}, \vec{c}$ are three non-Coplanar Vectors, then express $\vec{a}, \vec{b}, \vec{c}$ each in terms of $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$, $\vec{a}\times\vec{b}$.

(b) Prove that:
$$\vec{a}\times[\vec{b}\times(\vec{c}\times\vec{d})] = (\vec{b}\cdot\vec{d})(\vec{a}\cdot\vec{c}) - (\vec{b}\cdot\vec{c})(\vec{a}\times\vec{d})$$
$$= [\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - (\vec{a}\cdot\vec{b})(\vec{c}\times\vec{d})$$

4 (a) Define Vector Product of four vectors. Establish the ~~a~~ formula for $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$

(b) Define Scalar Product of four Vectors and establish a formula for $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$

5 (a) Define moment Vector of a force about a point and about a line. Prove that the moment vector ~~abou~~ about any point is independent of the choice of the point P on the line of action of force.

(b) Find the moment of a force $\vec{F} = 2\hat{i} + 9\hat{j} + 7\hat{k}$ acting at the point $5\hat{i} - \hat{j}$ about the point $\vec{i} - 2\vec{j} + 5\vec{k}$.

6 (a) A rigid body rotates with an angular speed of 2 radians per second about an axis passing through the point (0, 1, 2) and (1, 3, −2). Find the velocity of the point (3, 6, 4) of the rigid body.

(b) A particle is acted on by a constant force $4\vec{i} + 3\vec{j}$ and $3\vec{i} + 2\vec{j}$ is displaced from the point $\vec{i} + 2\vec{j}$ to $5\vec{i} + 4\vec{j}$. Find the total work done by the force.

7 (a) Define gradient of a scalar point function. Prove that the necessary and sufficient condition for a scalar point function ϕ to be constant is that $\nabla\phi = 0$

(b) Define curl of a vector point function and prove that

$$\text{curl}(\phi\vec{A}) = \phi \text{ curl } \vec{A} + (\text{grad } \phi) \times \vec{A}$$

where ϕ and $\vec{A}$ are continuously differentiable scalar and vector point functions respectively.

8 (a) Define Divergence of a vector point function and prove that $\text{div}(\phi\vec{A}) = \phi \text{ div } \vec{A} + (\text{grad } \phi) \cdot \vec{A}$

(b) Prove that $\text{div}(r^n \vec{r}) = (n+3) r^n$

9 (a) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an ~~a~~ arbitrary point in the plane of the forces together with a couple.

9 (b) Obtain the equation to the line of action of the resultant of a system of coplanar forces.

10 (a) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body.

(b) Three forces P, Q, R acting along the sides of the triangle formed by the lines $x+y=1$, $y-x=1$ and $y=2$. Find the equation to the line of action of the resultant.

11 (a) State and prove the principle of Virtual Work for any system of forces in one plane

(b) State and prove the converse of principle of Virtual Work for any system of coplanar forces.

12 (a) Enumerate the nature of forces which may be omitted in forming the equation of Virtual work, giving reasons.

(b) The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length l and l'. If T and T' be the tensions in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

13 (a) Prove that the tension at P of the catenary is equal to the weight of the string whose length is the vertical distance between P and the directrix OX.

(b) A uniform chain of length l is to be suspended from two points A, B in the same horizontal line so that either terminal tension is n times that at the lowest point. Prove that the span

$$AB = \frac{l}{\sqrt{n^2-1}} \log\left(n + \sqrt{n^2-1}\right)$$

14 (a) Establish the formula $T^2\mu =$ Constant, where the symbols have the usual significance.

(b) A particle starts with a velocity V and moves under a retardation equal to K times the space described. Prove that the space traversed before it comes to rest is equal to $\frac{V}{\sqrt{K}}$.

15 Find the expressions for

(i) radial velocity & radial accelaration

(ii) transverse velocity & tangential accl$\underline{n}$.

16. Find the expressions for

(i) tangential velocity and tangential accl$\underline{n}$

(ii) Normal Velocity and Normal accl$\underline{n}$.

17 (a) A particle describes an equiangular spiral $r = ae^{m\theta}$ with constant velocity. Find the components of the velocity and the accl$\underline{n}$ along the radius vector and perpendicular to it

(b) A point moves in a plane curve so that its tangential and normal accelarations are equal and the tangent rotates with constant angular velocity. Find the path.

18 (a) A particle moves in a plane with an accelaration which is always directed to

a fixed point O in the plane. Obtain the D.E. of the path.

(b) A particle moves in a path so that its accelaration is always directed to a fixed path and is equal to $\frac{\mu}{(\text{distance})^2}$, show that the path is a conic section and distinguish between the three cases that arise.

19 State and explain Kepler's Laws of planetary motion. Also deduce the laws from Newton's Law of gravitation.

20 Solve:

(i) $y - x\frac{dy}{dx} = a(y^2 + \frac{dy}{dx})$

(ii) $\frac{dy}{dx} = e^{x-y} - 1$

(iii) $(1+e^{x/y})\,dx + e^{x/y}(1 - x/y)\,dy = 0$

(iv) $\frac{dy}{dx} = \frac{2x - 6y + 7}{x - 3y + 4}$

(v) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$.

(vi) $\frac{dy}{dx} + xy = x^3y^3$

(VII) $y = 2px + p^3$, $p \equiv \frac{dy}{dx}$

(VIII) $x + \frac{p}{\sqrt{1+p^2}} = a$, $p \equiv \frac{dy}{dx}$

(ix) $(px - y)(py + x) = h^2 p$

(x) Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c=0$

(XI) Find the orthogonal trajectories of $r^n \sin n\theta = a^n$.

(XII) $(D^2+D)\,y = 1 + \cosh x$

(XIII) $(D^2+a^2)\,y = 24\cos x$

(XIV) $(D^2+a^2)\,y = \sec ax$

(XV) $(D^2-2D+1)\,y = x e^x \sin x$

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